Quantum nonlocality of massive qubits in a moving frame

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We perform numerical tests on quantum nonlocality of two-level quantum systems (qubits) observed by a uniformly moving observer. Under a suitable momentum setting, the quantum nonlocality of two-qubit nonmaximally entangled states could be weakened drastically by the Lorentz transformation allowing for the existence of local-hidden-variable models, whereas three-qubit genuinely entangled states are robust. In particular, the generalized GHZ state remains nonlocal under arbitrary Wigner rotation and the generalized W state could admit local-hidden-variable models within a rather narrow range of parameters.

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I. INTRODUCTION

Most issues in quantum information science focussed on problems that elucidate and exemplify the difference between classical and quantum mechanics within non-relativistic context. However, in recent years, it has been realized that fundamental notions in quantum information theory undergo substantial revision under relativistic settings. The theory of relativity requires a physical quantity to be Lorentz-invariant. Thus, there were several attempts [1] to develop notions in quantum information like measures for quantum entanglement and quantum teleportation fidelity and so forth within relativistic settings so that they remain Lorentz invariant. There is generally no consensus on such modifications needed in this respect [2]. Clearly, the search for Lorentz-invariant properties in quantum systems is quite a non-trivial task.

Bell's inequality [3, 4] can be regarded as a hybrid of relativity and quantum mechanics. On one hand, its derivation adopts the assumption of locality, an important feature in relativity. On the other hand, an observable under measurements is described by the Hermitian operator based on the standard techniques in quantum mechanics to account for a measurable physical quantity. One may observe that Lorentz-invariance, an important feature in relativity, is explicitly missing in Bell's inequality. This naturally leads to the question on how quantum nonlocality adapts itself when Lorentz invariance is taken into account. This is principal motivation behind our current work.

To this end, we consider the Wigner rotation [5], a significant relativistic effect related to the Lorentz transformation. Before proceeding further, there is an important consideration: viz. the relativistic counterpart of the spin operator in quantum mechanics [6–8]. However, as we show in Sec. III, insofar as quantum nonlocality is concerned, we may somehow neglect this consideration for all practical purposes for qubit systems.

It is convenient to recast the eigenstate of quantum systems as a product of "momentum state" and "spin state": $|\Psi\rangle = |\psi_{\rm mom}\rangle \otimes |\phi_{\rm spin}\rangle$. Under relativistic settings, there is mixing between the spin and momentum parts. Indeed, the relativistic quantum nonlocality of two- and three-qubit maximally entangled spin states have been explored in Refs. [9]. In these references, the momentum state is essentially a product state and the Wigner rotation is equivalent to a local unitary transformation of the spin state. Under such condition, quantum nonlocality is anticipated to be Lorentz-invariant as well as frame-independent. Authors in Ref. [10] also considered the two-qubit case where the momentum state is entangled and showed that the partial entropy drastically decreases with respect to large Wigner angles. In Ref. [11], the authors considered an alternative three-qubit momentum setting with two genuinely entangled spin states (GHZ and W states) using a version of multi-partite concurrence and derived general conditions which have to be met for any classification of multi-partite entanglement to be Lorentz-invariant.

In this work, we consider more general situations using Bell's inequality. We first consider the two-qubit case in which the spin part is a generalized GHZ state. We then investigate two inequivalent classes of genuine three-qubit entangled states: generalized GHZ and W states. Different momentum settings are also compared and dis-

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cussed.

The paper is organized as follows. In Sec. II, we provide a definition of the Wigner rotation and discuss its effect on multi-partite quantum states. In Sec. III, we discuss the possible candidates for relativistic spin operators and their relationship to the usual Pauli operator. In Sec. IV, we present our main results on relativistic quantum nonlocality of two and three qubits. We end the paper with a summary.

II. THE WIGNER ROTATION

The Wigner rotation can be understood algebraically as the consequence of the non-associativity of the relativistic addition of velocities (Einstein's addition). In group theory, the Wigner rotation is rigorously defined as the little group [5]:

$$W(\Lambda,p)=L^{-1}(\Lambda p)\Lambda L(p), \tag{1}$$

where $L(p)$ denotes a standard Lorentz transformation that transforms the particle from rest to four-momentum p^μ, namely,

$$p^\mu=L^\mu_\nu(p)k^\nu,\ \ k^\nu=(m,0,0,0). \tag{2}$$

[In this section, we use notations similar to those in Ref. [12] and adopt natural units making $c=1$.]

The one-particle state with momentum $\vec{p}$ and spin σ can be expressed by

$$|\vec{p},\sigma\rangle\equiv a^\dagger(\vec{p},\sigma)|\text{vac}\rangle, \tag{3}$$

with $|\text{vac}\rangle$ the Lorentz-invariant vacuum state and $a^\dagger(\vec{p},\sigma)$ the creation operator transforming under the unitary $U(\Lambda)$ with the following rule:

$$U(\Lambda)a^\dagger(\vec{p},\sigma)U^{-1}(\Lambda)=\sum_{\sigma'}D^j_{\sigma'\sigma}(W(\Lambda,p))a^\dagger(\vec{p}_\Lambda,\sigma'), \tag{4}$$

where $D^j_{\sigma'\sigma}(W(\Lambda,p))$ are elements of the $(2j+1)$-dimensional representation of the Wigner rotation $W(\Lambda,p)$, and $\vec{p}_\Lambda$ is the spatial component of the transformed four-momentum Λp. Thus, when the observer is moving at a certain constant velocity $\vec{v}$, the one-particle state (3) is transformed further to

$$U(\Lambda)|\vec{p},\sigma\rangle=\sum_{\sigma'}D^j_{\sigma'\sigma}(W(\Lambda,p))|\vec{p}_\Lambda,\sigma'\rangle. \tag{5}$$

We can interpret (5) in the following manner: The observer is at rest and the frame which contains the particle in the state $|\vec{p},\sigma\rangle$ is moving. Then the Wigner angle arises after three steps: (i) a particle with spin σ is created at rest from vacuum, we get the state $|0,\sigma\rangle$; (ii) the frame which contains the particle moves in the velocity $\vec{u}=\vec{p}/p^0$, thus the one-particle state $|0,\sigma\rangle$ is transformed to $|\vec{p},\sigma\rangle$ by $U(L(p))$; (iii) the frame further moves in the velocity $\vec{v}$, i.e. the second Lorentz transformation $U(\Lambda)$ acts on state $|\vec{p},\sigma\rangle$. According to (1), two successive Lorentz transformations $L(p)$ and Λ are equal to a single $L(\Lambda p)$ combined with the Wigner rotation W. Therefore, the scenario of a moving particle observed in the moving frame is elegantly equivalent to that of an observer at rest who observes a successively Lorentz-transformed particle.

The multipartite state is expressed by

$$|\vec{p}_1,\sigma_1;\vec{p}_2,\sigma_2\cdots\rangle\equiv a^\dagger(\vec{p}_1,\sigma_1)a^\dagger(\vec{p}_2,\sigma_2)\cdots|\text{vac}\rangle, \tag{6}$$

As observed in a moving frame, each $a^\dagger(\vec{p}_k,\sigma_k)$ transforms according to (4). The Lorentz-transformed state is found to be

$$\begin{aligned}U(\Lambda)&|\vec{p}_1,\sigma_1;\vec{p}_2,\sigma_2\cdots\rangle\\&=\sum_{\sigma_1'\sigma_2'\cdots}D^{j_1}_{\sigma_1'\sigma_1}(W(\Lambda,p_1))D^{j_2}_{\sigma_2'\sigma_2}(W(\Lambda,p_2))\cdots\\&\qquad\times|\vec{p}_{1\Lambda},\sigma_1';\vec{p}_{2\Lambda},\sigma_2'\cdots\rangle.\end{aligned} \tag{7}$$

For the sake of convenience later, we can also separate momentum $\vec{p}_k$ explicitly from spin σ_k so that the multipartite state is effectively a product of momentum and spin. For instance, Eq. (7) becomes

$$\begin{aligned}U(\Lambda)&|\vec{p}_1,\vec{p}_2\cdots\rangle\otimes|\sigma_1,\sigma_2\cdots\rangle\\&=|\vec{p}_{1\Lambda},\vec{p}_{2\Lambda}\cdots\rangle\\&\quad\otimes\sum_{\sigma_1'\sigma_2'\cdots}D^{j_1}_{\sigma_1'\sigma_1}(W(\Lambda,p_1))D^{j_2}_{\sigma_2'\sigma_2}(W(\Lambda,p_2))\cdots\\&\qquad\times|\sigma_1',\sigma_2'\cdots\rangle.\end{aligned} \tag{8}$$

Note that if we focus on the spin state $|\sigma_1,\sigma_2\cdots\rangle$, the Wigner rotation can be regarded as a local unitary transformation since the little group $W(\Lambda,p_k)$ and its the representation $D^{j_k}(W(\Lambda,p_k))$ are unitary.

The little group $W(\Lambda,p)$ for massive particles is $SO(3)$. Representations of this little group have been systematically studied using the method of induced representations [13]. In the following context, we restrict ourselves to two-level particles (qubits). The two-dimensional representations $D^{1/2}(W(\Lambda,p_k))$ are employed and the generators are Pauli matrices $\vec{s}$. For our purpose, the particles are moving with velocity $\vec{u}_k$ in the yz-plane and the observer is moving with velocity $\vec{v}$ along the x-axis, then the two-dimensional representation is found to be [12]

$$D^{j_k=\frac{1}{2}}(W(\Lambda,p_k))=\cos\frac{\Omega(\vec{u}_k,\vec{v})}{2}+i\vec{s}\cdot\vec{n}_k\sin\frac{\Omega(\vec{u}_k,\vec{v})}{2}, \tag{9}$$

with $\vec{n}_k=\vec{v}\times\vec{u}_k/(|\vec{v}|\cdot|\vec{u}_k|)$. The Wigner angle $\Omega(\vec{u}_k,\vec{v})$ is calculated by

$$\Omega(\vec{u}_k,\vec{v})=\arctan\frac{\sinh\xi\sinh\zeta}{\cosh\xi+\cosh\zeta}, \tag{10}$$

with

$$\cosh\xi=\frac{1}{\sqrt{1-|\vec{u}_k|^2}},\ \ \cosh\zeta=\frac{1}{\sqrt{1-|\vec{v}|^2}}. \tag{11}$$

Note that the Wigner angle is zero as $|\vec{u}_k| = 0$ or $|\vec{v}| = 0$, and goes up to $\pi/2$ as $|\vec{u}_k|$ and $|\vec{v}|$ approach the speed of light $c = 1$.

Suppose the initial state can be rewritten as a product of momentum and spin states, i.e.,

$$|\Psi\rangle = |\psi_{\rm mom}\rangle \otimes |\phi_{\rm spin}\rangle, \tag{12}$$

or

$$\rho = \rho_{\rm mom} \otimes \rho_{\rm spin}, \tag{13}$$

with

$$\rho_{\rm mom} = |\psi_{\rm mom}\rangle\langle\psi_{\rm mom}|, \quad \rho_{\rm spin} = |\psi_{\rm spin}\rangle\langle\psi_{\rm spin}|. \tag{14}$$

Here $\rho_{\rm mom}$ and $\rho_{\rm spin}$ could be entangled states. To see the dependence of the Wigner rotation on momentum, the state observed by the moving observer becomes

$$\rho' = U(\Lambda)\rho U(\Lambda)^{-1}. \tag{15}$$

This transformed state may not generally be in the form of product $\rho'_{\rm mom} \otimes \rho'_{\rm spin}$ and the reduced spin state

$$\rho'_{\rm spin} = {\rm tr}_{\rm mom}\rho' \tag{16}$$

is a mixed state. Since it has been proved that there exist some mixed states that bear local-hidden-variable models [14], the Wigner rotation of quantum states is non-trivial in the study of quantum nonlocality.

III. THE RELATIVISTIC OBSERVABLES

There have been several proposals for the relativistic spin operator [7, 8], derived under very different physical requirements and not mutually equivalent to one another. For instance, deriving from relativistic center of mass, one obtains a typical relativistic spin operator [7]:

$$\hat{a} = \frac{(\sqrt{1-|\vec{w}_k|^2}\vec{a}^{\perp} + \vec{a}^{\parallel}) \cdot \vec{s}}{\sqrt{1+(\vec{w}_k \cdot \vec{a})^2 - |\vec{w}_k|^2}}, \tag{17}$$

where $\vec{a} = \vec{a}^{\perp} + \vec{a}^{\parallel}$ is the measuring direction with components $\vec{a}^{\perp}$ and $\vec{a}^{\parallel}$ respectively perpendicular and parallel to the direction of the composite velocity $\vec{w}_k$ of $\vec{u}_k$ and $\vec{v}$ [15].

One can verify that eigenvalues of this relativistic spin operator are ± 1. In fact, by rotating along a certain direction, any normalized relativistic spin operator which is in the form of linear combination of Pauli matrices coincides with the nonrelativistic spin operator $\vec{a} \cdot \vec{s}$. To see this clearly, let us take another look at the operator (17). As mentioned above, this is a relativistic spin operator measured in the direction $\vec{a}$. From the nonrelativistic viewpoint, (17) is effectively a nonrelativistic spin operator $\vec{a'} \cdot \vec{s}$ measured in the direction $\vec{a'}$. In principle, there must be a unitary U that rotates $\vec{a'}$ to $\vec{a}$, i.e.,

$$\vec{a'} \cdot \vec{s} = U\vec{a} \cdot \vec{s}\, U^{-1}. \tag{18}$$

This is crucial in the relativistic quantum nonlocality. This fact implies that instead of the relativistic ones we can adopt the nonrelativistic spin operators to obtain the quantum violation of Bell's inequality. In other words, if one uses the relativistic spin operators measured along a certain set of directions to calculate the quantum value, then one can obtain the same value by using the nonrelativistic ones, with measuring directions modified by some unitary U.

The M-setting N-qubit Bell inequality I_N can be written in the correlational form:

$$I_N = \sum_{i_1, i_2, \ldots, i_N = 0}^{M} T_{i_1 i_2 \ldots i_N} Q_{i_1 i_2 \ldots i_N} \leq 1, \tag{19}$$

where $T_{i_1 i_2 \ldots i_N}$ are coefficients and $Q_{i_1 i_2 \ldots i_N}$ are correlation functions defined by

$$Q_{i_1 i_2 \ldots i_N} = {\rm tr}[\rho_{\rm spin}\vec{a}_{i_1} \cdot \vec{s} \otimes \vec{a}_{i_2} \cdot \vec{s} \otimes \cdots \vec{a}_{i_N} \cdot \vec{s}], \tag{20}$$

with $\vec{a}_{i_k}$ the i_k-th measuring direction of the k-th qubit, if $i_k \neq 0$. When $i_k = 0$, this means no measurement is performed on the k-th qubit, thus the correlation function is modified by substituting the identity $\mathbb{1}$ for spin operator $\vec{a}_{i_k} \cdot \vec{s}$.

Following the analysis above, the quantum value I_N with respect to some settings $\{\vec{a}_{i_1}, \vec{a}_{i_2}, ..., \vec{a}_{i_N}\}$ using relativistic spin operators is the same to that with respect to modified settings $\{\vec{a'}_{i_1}, \vec{a'}_{i_2}, ..., \vec{a'}_{i_N}\}$ using nonrelativistic ones. Therefore, as far as qubit systems are concerned, the usual Pauli operator $\vec{s}$ is adequate to study the quantum nonlocality.

IV. RELATIVISTIC QUANTUM NONLOCALITY OF TWO AND THREE QUBITS

Recalling the analysis in Sec. II, the observed spin state for the moving observer is a Lorentz-transformed one:

$$\rho_{\rm spin} \to \rho'_{\rm spin}. \tag{21}$$

To test this state, we can readily use the Bell inequalities which have been employed to study nonlocality in nonrelativistic quantum mechanics, since the derivation of Bell inequalities is inherently in the relativistic treatment.

For two qubits, we consider the CHSH inequality [4]

$$I_2 = \frac{1}{2}\Big(Q_{11} + Q_{12} + Q_{21} - Q_{22}\Big) \leq 1, \tag{22}$$

with $Q_{ij} = {\rm tr}[\rho\vec{a}_i \cdot \vec{s} \otimes \vec{a}_j \cdot \vec{s}]$. The initial state we consider is

$$|\Psi_1\rangle = \Big(\cos\theta_m|\vec{p}_1, \vec{p}_2\rangle + \sin\theta_m|-\vec{p}_1, -\vec{p}_2\rangle\Big) \otimes \Big(\cos\theta_s|00\rangle + \sin\theta_s|11\rangle\Big), \tag{23}$$

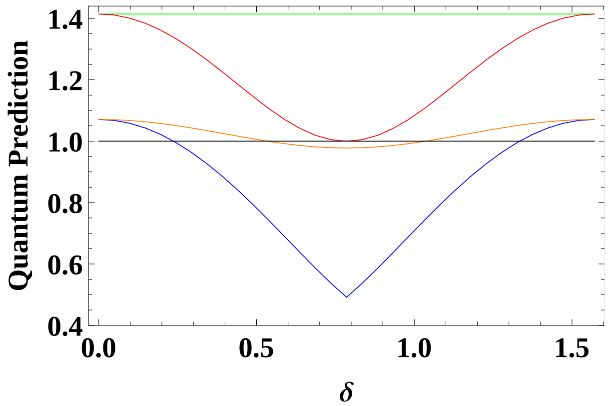

FIG. 1: (Color online) The variation of quantum values I_2 with respect to the Wigner rotations of the initial state (23) in different momentum settings (24) and (25). The red and blue curves correspond to parameters $\{\theta_m, \theta_s\}$ taking $\{\frac{\pi}{4}, \frac{\pi}{4}\}$ and $\{\frac{\pi}{4}, \frac{\pi}{16}\}$ in (24), respectively; and the green and orange curves correspond to parameters $\{\theta_m, \theta_s\}$ taking $\{\frac{\pi}{4}, \frac{\pi}{4}\}$ and $\{\frac{\pi}{4}, \frac{\pi}{16}\}$ in (25), respectively.

with two different momentum settings

$$\mathrm{p}_1 = -\mathrm{p}_2 = (0, 0, 1), \tag{24}$$

and

$$\mathrm{p}_1 = \mathrm{p}_2 = (0, 0, 1). \tag{25}$$

Here $\mathrm{p}_k = \vec{p}_k/|\vec{p}_k|$ indicates the moving direction of the k-th qubit (i.e., p_k is proportional to $\vec{u}_k$ in Sec. II). In the former setting, two qubits are in the superposition of moving in opposite directions, while in the latter they are in the superposition of moving in the same direction (z-axis or the opposite). For simplicity, we take $|\vec{p}_1| = |\vec{p}_2|$.

As the experimentalist moves in the opposite x-axis (or equivalently, the experimentalist stays in his rest frame and the qubits system moves in the x-axis), the observed state is a transformed one by the Wigner rotation of (23). Fig. 1 shows the quantum values of I_2 with respect to various Wigner angles. For the momentum setting (24) (see the red and blue curves), there would be a region where local-hidden-variable models are admitted, unless either the entanglement degree of the momentum part is small or the spin state is maximally entangled. For the momentum setting (25) (see the green and orange curves), we have similar results, except that (i) the effect of the Wigner rotation is weaker than that in (24), and (ii) the Wigner rotation does not affect the quantum values when the spin state is maximally entangled.

For three qubits, we use the following inequality proposed in Ref. [16]:

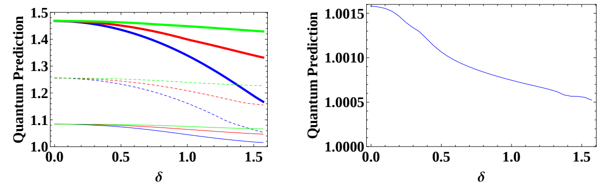

FIG. 2: (Color online) The variation of quantum values I_3 with respect to the Wigner rotations of the initial state (27) in the momentum setting (29). Left: The colors $\{\text{blue}, \text{red}, \text{green}\}$ correspond to parameter $\theta_m = \{\frac{\pi}{4}, \frac{\pi}{8}, \frac{\pi}{16}\}$, respectively; the line styles $\{\text{thick}, \text{dashed}, \text{thin}\}$ correspond to parameter $\theta_s = \{\frac{\pi}{4}, \frac{\pi}{8}, \frac{\pi}{16}\}$, respectively. Right: The curve corresponds to $\{\theta_m, \theta_s\} = \{\frac{\pi}{4}, \frac{\pi}{128}\}$.

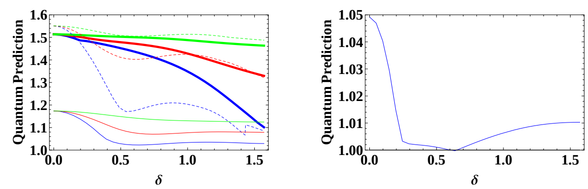

FIG. 3: (Color online) The variation of quantum values I_3 with respect to the Wigner rotations of the initial state (28) in the momentum setting (29). Left: The colors $\{\text{blue}, \text{red}, \text{green}\}$ correspond to parameter $\theta_m = \{\frac{\pi}{4}, \frac{\pi}{8}, \frac{\pi}{16}\}$, respectively; the line styles $\{\text{thick}, \text{dashed}, \text{thin}\}$ correspond to parameter pair $\{\theta_s, \phi_s\} = \left\{\{\arccos\frac{1}{\sqrt{3}}, \frac{\pi}{4}\}, \{\frac{7\pi}{16}, \frac{\pi}{4}\}, \{\frac{7\pi}{16}, \frac{\pi}{16}\}\right\}$, respectively. Right: The curve corresponds to $\{\theta_m, \theta_s, \phi_s\} = \{\frac{\pi}{4}, \frac{15\pi}{32}, \frac{\pi}{32}\}$.

$$\begin{aligned} I_3 = \frac{1}{3}\Big(& -Q_{111} + Q_{221} + Q_{212} + Q_{122} - Q_{222} - Q_{110} \\ & -Q_{120} - Q_{210} - Q_{101} - Q_{102} - Q_{201} - Q_{011} \\ & -Q_{012} - Q_{021} + Q_{200} + Q_{020} + Q_{002}\Big) \le 1, \end{aligned} \tag{26}$$

with $Q_{ijk} = \mathrm{tr}[\rho\vec{a}_i \cdot \vec{s} \otimes \vec{a}_j \cdot \vec{s} \otimes \vec{a}_k \cdot \vec{s}]$, and subscript "0" indicating that no measurement is performed on the corresponding qubit. For instance, $Q_{ij0} = \mathrm{tr}[\rho\vec{a}_i \cdot \vec{s} \otimes \vec{a}_j \cdot \vec{s} \otimes \mathbb{1}]$. The initial states we consider are

$$\begin{aligned} |\Psi_2\rangle = & \Big(\cos\theta_m|\vec{p}_1, \vec{p}_2, \vec{p}_3\rangle + \sin\theta_m|-\vec{p}_1, -\vec{p}_2, -\vec{p}_3\rangle\Big) \\ & \otimes\Big(\cos\theta_s|000\rangle + \sin\theta_s|111\rangle\Big), \end{aligned} \tag{27}$$

$$\begin{aligned} |\Psi_3\rangle = & \Big(\cos\theta_m|\vec{p}_1, \vec{p}_2, \vec{p}_3\rangle + \sin\theta_m|-\vec{p}_1, -\vec{p}_2, -\vec{p}_3\rangle\Big) \\ & \otimes\Big(\sin\theta_s\cos\phi_s|001\rangle + \sin\theta_s\sin\phi_s|010\rangle \\ & + \cos\theta_s|100\rangle\Big), \end{aligned} \tag{28}$$

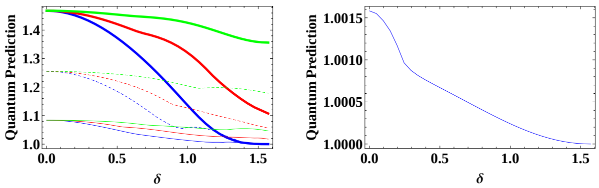

FIG. 4: (Color online) The variation of quantum values I_3 with respect to the Wigner rotations of the initial state (27) in the momentum setting (30). Left: The colors {blue, red, green} correspond to parameter $\theta_m = \{\frac{\pi}{4}, \frac{\pi}{8}, \frac{\pi}{16}\}$, respectively; the line styles {thick, dashed, thin} correspond to parameter $\theta_s = \{\frac{\pi}{4}, \frac{\pi}{8}, \frac{\pi}{16}\}$, respectively. Right: The curve corresponds to $\{\theta_m, \theta_s\} = \{\frac{\pi}{4}, \frac{\pi}{128}\}$.

with

$$\begin{aligned} \mathrm{p}_1 &= (0, 0, 1), \\ \mathrm{p}_2 &= (0, \sqrt{3}/2, -1/2), \\ \mathrm{p}_3 &= (0, -\sqrt{3}/2, -1/2), \end{aligned} \tag{29}$$

with $\mathrm{p}_k = \vec{p}_k/|\vec{p}_k|$ and $|\vec{p}_1| = |\vec{p}_2| = |\vec{p}_3|$, similar to the two-qubit case. Note that the spin states of (27) and (28) belong to two inequivalent classes of genuine three-qubit entangled states: generalized GHZ and W states.

Fig. 2 (left) and Fig. 3 (left) show the quantum values of I_3 with respect to various Wigner angles for several typical parameters taken in the initial states (27) and (28), respectively. It is found that for given parameters in the spin states, the Wigner rotation weakens quantum nonlocality the most as the momentum states are maximally entangled. Thus in Fig. 2 (right) we further take $\theta_m = \frac{\pi}{4}$ (maximally entangled momentum state) and $\theta_s = \frac{\pi}{128}$ (close to the separable spin state $|000\rangle$), and then find that quantum values are still larger than 1.

In Fig. 3 (left) we consider three types of generalized W states: (i) all three components $\{|001\rangle, |010\rangle, |100\rangle\}$ have equal weights (see three thick curves), (ii) one component is relatively smaller than the others (see three dashed curves), and (iii) one component is relatively larger than the others (see three thin curves). Among them, type (ii) is close to bi-separable state $|0\rangle \otimes (|01\rangle + |10\rangle)/\sqrt{2}$, type (iii) is close to tri-separable state $|001\rangle$. In Fig. 3 (right) we further take $\theta_m = \frac{\pi}{4}$, $\theta_s = \frac{15\pi}{32}$ and $\phi_s = \frac{\pi}{32}$, and then find that quantum values are larger than 1 for almost the whole region. The minimum is approximately 0.9997 near the point $\delta \approx 0.64$.

Moreover, the GHZ and W states as the spin states in (27) and (28) are the only two classes of genuine three-qubit entangled states [17]; the other genuine entangled states are local unitary (LU) equivalent to one of them. Therefore, it is reasonable to draw a conclusion that quantum nonlocality of genuine three-qubit entangled states is robust against the Lorentz transformation.

We must stress that this conclusion is drawn by a proper selection of momentum magnitude and directions. Under the Lorentz transformation, the inevitable coupling of momentum and spin results in the relativistic quantum nonlocality of spin states also sensitively depending on details of momentum. To see this, let us change momentum directions in (27) to

$$\mathrm{p}_1 = \mathrm{p}_2 = \mathrm{p}_3 = (0, 0, 1). \tag{30}$$

The corresponding quantum values are shown in Fig. 4. It is obvious that the curves are different from those in Fig. 2.

However, it is also interesting that in this momentum setting the Lorentz-transformed state still remains nonlocal under the Lorentz transformation. Whether other types of entangled momentum and spin parts (for instance, partially entangled states) remain nonlocal under the Lorentz transformation, and if not, how the nonlocality is weakened with respect to various parameters in transformations are also intriguing questions subsequently.

V. SUMMARY

To investigate relativistic quantum nonlocality, we have taken into account the composite motion of both spins and the observer. This motion is non-trivial and will cause the Wigner rotation of particle states. We have shown that quantum nonlocality of two-qubit states could be drastically weakened if the entanglement degree is not maximal. In the three-qubit case, however, we have shown that quantum nonlocality of genuinely entangled states remains nonlocal with respect to almost arbitrary Wigner angles. Moreover, we have also pointed out that one should carefully consider the details of particle momentum, since spin is inevitably coupled to momentum under the Lorentz transformation.

Here are a few words before ending the paper. Physically, the momentum setting in (24) or (29) describes a particle that decays into several subparticles traveling uniformly in space with two possibilities. An alternative momentum setting as taken in (25) or (30) describes a bunch of beam in which particles travel in the same direction, positive or negative z-axis. These two settings may be more feasible than the others in the experimental state preparation in testing relativistic quantum nonlocality.

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